REAL OPTION VALUE

CHAPTER 14 INCENTIVE OPTIONS

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Incentive options can be viewed using the toolkit implicit in previous chapters of real payoff diagrams, entry and exit options, and perpetual American puts and calls. Incentive options may be granted (or required by) governments to encourage early investment in "desirable" projects such as renewable energy facilities, infrastructure investments like roads, bridges and other transportation, and in general public-private partnerships governing new facilities like schools, hospitals, and recreation areas.

These incentive options are classified as (i) proportional revenue (or price and/or quantity) subsidies, where the market price and/or the quantity of production is uncertain or low, but the subsidy is proportional to the quantity produced (ii) supplementary revenue (or price and/or quantity) subsidies, where the market price and/or the quantity of production and/or the exogenous subsidy is uncertain (iii) revenue floors and ceilings, where the subsidy is related over time to the actual quantities produced or market prices. Examples of (i) are so-called Feed-in-Tariffs "FiT" which are fixed amount subsidies per unit production, (ii) renewable "green" certificates, which have an uncertain value but are usually allocated per unit of production, and (iii) government minimum revenue guarantees, sometimes accompanied by maximum revenue ceilings.

In addition, governments provide incentives for free or at low cost (sport stadiums, concessions, priority access, protection through tariffs, quotas or security) in order to encourage "desirable" activities, or investment cost reliefs, consisting of direct grants and soft loans, tax credits or excess depreciation, which are not directly considered here, except in examining sensitivities of thresholds and real option value to changes in investment costs or taxation. Some of these incentives can be evaluated in terms of the real option value compared to that paid to the government (taxes, concession and user fees and royalties) weighted against the immediate or eventual cost for the government. Also it is interesting to study the effect of incentives on the real option value, and on the threshold that justifies immediate investment, as price, quantity and subsidies change. Who gets/gives what, when, how, and why are almost always critical considerations in incentive options.

14.3 Revenue Floors & Ceilings

The real American collar option for a certain asset confines the effective price within specified floor (lower) and ceiling (upper) limits. Acting as a risk moderator, the collar offers protection against the adversity from extreme falls in the output price or rises in the procurement price while simultaneously extracting some incremental value from favourable prices. Consequently, the upside gains partially compensate the downside losses. Unlike financial options, real American perpetuities on specific projects are currently not obtainable from the market, but governments may be agreeable to grant and underwrite price limits in certain circumstances. The pursuance of an energy diversity goal may motivate governments to enact a policy that subsidizes renewable energy investors by guaranteeing a fixed price in the form of a contract-for-differences deal. Similarly, foreign investors are induced to locate in countries whose governments grant subsidized or preferential procurement prices for raw materials or energy. The role of these subsidies is to raise the investment option value and to reduce the investment threshold, which not only render an investment more attractive but also hasten its exercise.

In a real option framework there are several articles on the effect of a subsidy on the investment value and policy. Boomsma et al. (2012) evaluate energy subsidies. Adkins and Paxson (2015) consider permanent and retractable subsidies as do Boomsma and Linnerud (2015), but not revenue ceilings. Takashima et al. (2010) design a *public-private partnership (PPP)* deal involving government debt participation that incorporates a floor on the future maximum loss level, where the concessionaire has the right to sell back the project to the government whenever adverse conditions emerge. Armada et al. (2012) investigate a subsidy in the form of a perpetual put option on the output price with protection against adverse price movements.

Only *Adkins and Paxson (2016B)* consider perpetual collar options in PPPs. From a general model, separate price floor subsidies and price ceilings are specific examples of general collar options imposed on the active project value. A price collar option contributes both positively and negatively to the active project value, and also to the real option value of an opportunity to invest in such a project.

14.3.1 Fundamental Model

For a firm in a monopolistic setting confronting a single source of uncertainty due to output price[1] variability, and ignoring operating costs and taxes, the opportunity to invest in an irretrievable project at

[1] This model can easily be altered to involve only quantity (Q) uncertainty, for toll roads with stochastic traffic and fixed tolls, where R=P*Q.

cost K depends solely on the price evolution, which is specified by the geometric Brownian motion process:

$$\mathrm{d}P = \alpha P \mathrm{d}t + \sigma P \mathrm{d}W \tag{1}$$

where α denotes the expected price risk-neutral drift, σ the price volatility, and $\mathrm{d}W$ an increment of the standard Wiener process. Using contingent claims analysis, the option to invest in the project $F(P)$ follows the risk-neutral valuation relationship:

$$\tfrac{1}{2}\sigma^2 P^2 \frac{\partial^2 F}{\partial P^2} + (r-\delta) P \frac{\partial F}{\partial P} - rF = 0 \tag{2}$$

where $r > \alpha$ denotes the risk-free interest rate and $\delta = r - \alpha$ the rate of return shortfall. The generic solution to (2) is:

$$F(P) = A_1 P^{\beta_1} + A_2 P^{\beta_2} \tag{3}$$

where A_1, A_2 are to be determined generic constants and β_1, β_2 are, respectively, the positive and negative roots of the fundamental equation, which are given by:

$$\beta_1, \beta_2 = \left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right) \pm \sqrt{\left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{4}$$

In (3), if $A_2 = 0$ then F is a continuously increasing function of P and represents an American perpetual call option, Samuelson (1965), while if $A_1 = 0$ then it is a decreasing function and represents a put option, Merton (1973).

In the absence of other forms of optionality and a fixed output volume Q, a firm optimally invests when the value matching relationship linking the call option value and the net proceeds $PQ/\delta - K$ holds:

$$A_0 P^{\beta_1} = PQ/\delta - K \,. \tag{5}$$

Following standard methods, the without-collar optimal price threshold level triggering investment $\hat{P}_0$ is:

$$\hat{P}_0 = \frac{\beta_1}{\beta_1 - 1} \frac{\delta}{Y} K \;, \tag{6}$$

and the value function is:

$$F_0(P) = \begin{cases} = \dfrac{K}{\beta_1 - 1}\left(\dfrac{P}{\hat{P}_0}\right)^{\beta_1} & \text{for } P < \hat{P}_0 \\ = \dfrac{PQ}{\delta} - K & \text{for } P \geq \hat{P}_0, \end{cases} \tag{7}$$

with:

$$A_0 = \frac{\hat{P}_0^{1-\beta_1} Y}{\delta \beta_1} = \frac{K \hat{P}_0^{-\beta_1}}{\beta_1 - 1}. \tag{8}$$

14.3.2 Investment and Collar Option

Real Collar Option

A collar option is designed to confine the output price for an active project to a tailored range, by restricting its value to lie between a floor level P_L and a cap level P_H. Whenever the price trajectory falls below the floor, the received output price is assigned the value P_L, and whenever it exceeds the cap, it is assigned the value P_H. By restricting the price to this range, the firm is benefiting from receiving a price that never falls below P_L and is obtaining protection against adverse price movements, whilst at the same time, it is being forced never to receive a price exceeding P_H from sacrificing the upside potential. Protection against downside losses are mitigated in part by sacrificing upside gains. If as part of its subsidy policy, a government offers a firm a price collar times some output Q, the government compensates the firm by a positive amount equaling $(P_L - P)Q$ whenever $P < P_L$, but if the cap is breached and $P > P_H$, then the firm reimburses the government by the positive amount $(P - P_H)Q$. It follows that for an active project, the revenue accruing to the firm is given by $\pi_C(P) = \min\{\max\{P_L, P\} P_H\} \times Q$ and its value V_C is described by the risk-neutral valuation relationship:

$$\tfrac{1}{2}\sigma^2 P^2 \frac{\partial^2 V_C}{\partial P^2} + (r - \delta) P \frac{\partial V_C}{\partial P} - rV_C + \pi_C(P) = 0. \tag{9}$$

The relationship (9) and (2) are identical in form except for the revenue function.

The valuation of a with-collar active project is conceived over three mutually exclusive exhaustive regimes, I, II and III, specified on the P line, each with its own distinct valuation function. Regimes I, II and III are defined by $P \le P_L$, $P_L < P \le P_H$ and $P_H \le P$, respectively. Over Regime I, the firm is granted a more attractive fixed price P_L compared with the variable price P, but also possesses a call-style option to switch to the more favorable Regime II as soon as P exceeds P_L. This switch option increases in value with P and has the generic form AP^{β_1}, where A denotes a to be determined generic coefficient. Over Regime III, the firm is not only obliged to accept the less attractive fixed price P_H instead of P but also has to sell a put-style option to switch to the less favorable Regime II as soon as P falls below P_H. This switch option decreases in value with P and has the generic form AP^{β_2}. Over Regime II, the firm receives the variable price P, possesses a put-style option to switch to the more favorable Regime I as soon as P falls to P_L, but sells a call-style option to switch to the less favorable Regime III as soon as P attains P_H. If the subscript C denotes the with-collar arrangement, then after paying the investment cost, Model 7, the valuation function for the firm managing the active project, is:

$$V_C(P) = \begin{cases} \dfrac{P_L Q}{r} + A_{C11} P^{\beta_1} & \text{for } P < P_L \\ \dfrac{PQ}{\delta} + A_{C21} P^{\beta_1} + A_{C22} P^{\beta_2} & \text{for } P_L \le P < P_H \\ \dfrac{P_H Q}{r} + A_{C32} P^{\beta_2} & \text{for } P_H \le P. \end{cases} \quad (10)$$

In (10), a coefficient's first numerical subscript denotes the regime $\{1 = I, 2 = II, 3 = III\}$, while the second denotes a call if 2 or a put if 1. The coefficients A_{C11}, A_{C22} are positive because the firm owns the options and a switch is beneficial. In contrast, the A_{C21}, A_{C32} are negative because the firm is selling the options and is being penalized by the switch. The real collar is composed of a pair of both call and put options. The first pair facilitates switching back and forth between Regimes I and II, which results in the firm being advantaged, while the second pair facilitates switching back and forth between Regimes II and III, which results in the firm being disadvantaged.

A switch in either direction between Regimes I and II occurs when $P = P_L$. It is optimal provided the value-matching relationship:

$$\frac{P_L Q}{r} + A_{C12} P^{\beta_2} = \frac{PQ}{\delta} + A_{C21} P^{\beta_1} + A_{C22} P^{\beta_2} \tag{11}$$

and its smooth-pasting condition expressed as:

$$\beta_2 A_{C12} P^{\beta_2} = \frac{PQ}{\delta} + \beta_1 A_{C21} P^{\beta_1} + \beta_2 A_{C22} P^{\beta_2} \tag{12}$$

both hold when evaluated at $P = P_L$. Similarly, a switch in either direction between Regimes II and III occurs when $P = P_H$. It is optimal provided the value-matching relationship:

$$\frac{PQ}{\delta} + A_{C21} P^{\beta_1} + A_{C22} P^{\beta_2} = \frac{P_H Q}{r} + A_{C31} P^{\beta_1} \tag{13}$$

and its smooth-pasting condition expressed as:

$$\frac{PQ}{\delta} + \beta_1 A_{C21} P^{\beta_1} + \beta_2 A_{C22} P^{\beta_2} = \beta_1 A_{C31} P^{\beta_1} \tag{14}$$

both hold when evaluated at $P = P_H$. This reveals that:

$$\begin{aligned} A_{C11} &= \left[\frac{P_H Q}{P_H^{\beta_1}} - \frac{P_L Q}{P_L^{\beta_1}} \right] \times \frac{\left(r\beta_2 - r - \delta\beta_2 \right)}{\left(\beta_1 - \beta_2 \right) r\delta} > 0,\ A_{C21} = \frac{P_H Q \left(r\beta_2 - r - \delta\beta_2 \right)}{P_H^{\beta_1} \left(\beta_1 - \beta_2 \right) r\delta} < 0, \\ A_{C22} &= \frac{-P_L Q \left(r\beta_1 - r - \delta\beta_1 \right)}{P_L^{\beta_2} \left(\beta_1 - \beta_2 \right) r\delta} > 0,\ A_{C32} = \left[\frac{P_H Q}{P_H^{\beta_2}} - \frac{P_L Q}{P_L^{\beta_2}} \right] \times \frac{\left(r\beta_1 - r - \delta\beta_1 \right)}{\left(\beta_1 - \beta_2 \right) r\delta} < 0. \end{aligned} \tag{15}$$

Figure 7 shows the spreadsheet solution for Model 7, a concession with a floor and a ceiling, where Q=1.With these parameter values, the present value of a concession with P=6 currently is 150.00 with a put value of 29.98 and a call value of -41.61, for a total net concession value of 150+29.98-41.61=138.37. Other easy combinations are the same concession with only a put protection on the downside which would be worth 179.98, or an unfortunate concession with no downside protection but ceding to the concession grantor upside gains past P=10, which would be worth 150-41.61=108.39. Note that the differential equation (9) is solved, calculating the ROV deltas and gammas in B31 and B32.

Of course, these values are highly sensitive to changes in all of the parameter values, as illustrated in the Appendix.

Figure 7

	A	B	C	D
1			**ACTIVE PPP WITH COLLAR**	
2	**INPUT**			EQ
3	P	6.00		
4	K	100.00		
5	σ	0.25		
6	r	0.04		
7	δ	0.04		
8	PL	**4**		
9	PH	**10**		
10	**OUTPUT**			
11	ROV CALL	61.8978	IF(B3<B13,((B4/(B14-1))*(B3/B13)^B14),B12)	7
12	P/δ-K	50.0000	MAX(B3/B7-B4,0)	5
13	P^	9.4279	(B14/(B14-1))*B4*B7	6
14	β_1	1.7369	0.5-(B6-B7)/(B5^2)+SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
15	A0	2.7547	(B4*(B13^-B14))/(B14-1)	8
16	VC	**138.3688**		10
17	VC PV	**150.0000**	IF(B3<$B$8,$B$8/B6,IF(B3>$B$9,$B$9/B6,B3/B7))	
18	β2	-0.7369	0.5-(B6-B7)/(B5^2)-SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
19	AC21*P^β1	**-41.6129**	**B23*(B3^B14)**	
20	AC22*P^β2	**29.9818**	**B24*(B3^B18)**	
21	VC	138.3688	B17+B19+B20	
22	AC11	1.7862	($B$9/($B$9^B14)-$B$8/($B$8^B14))*(B26/B28)	15
23	AC21	-1.8520	($B$9/($B$9^B14))*(B26/B28)	15
24	AC22	112.2797	(-$B$8/($B$8^B18))*(B27/B28)	15
25	AC32	-439.16	($B$9/($B$9^B18)-$B$8/($B$8^B18))*(B27/B28)	15
26	[ ]	-0.0400	(B6*B18-B6-B7*B18)	15
27	( )	-0.0400	(B6*B14-B6-B7*B14)	15
28	{ }	0.0040	(B14-B18)*B6*B7	15
29				
30	ODE	0.0000	0.5*(B5^2)*(B3^2)*B32+(B6-B7)*B3*B31-B6*B16+B3	9
31	VC Δ	9.2711	1/B7+B14*B23*(B3^(B14-1))+B18*B24*(B3^(B18-1))	
32	VC Γ	-0.4136	B14*(B14-1)*B23*(B3^(B14-2))+B18*(B18-1)*B24*(B3^(B18-2))	
33	VC	IF(B3<$B$8,$B$8/B6+B22*(B3^B14),IF(B3>$B$9,$B$9/B6+B25*(B3^B18),B3/B7+B23*(B3^B14)+B24*(B3^B18)))		

Investment Option with a Collar

The with-collar optimal price threshold $\hat{P}_C$ triggering an investment lies between the floor and cap limits, $P_L \le \hat{P}_C \le P_H$. The floor limit holds because no optimal solution exists in its absence, that is for $\hat{P}_C < P_L$. $\hat{P}_C$ attains a minimum of $P_L = rK / Q$ and a maximum of $\hat{P}_0$ for $P_L = 0$, so the introduction of a price floor always produces at least an hastening of the investment exercise and never its postponement. The cap limit holds because of the absence of any effective economic benefit from exercising at a price exceeding the cap. Initially the price can be presumed to be near zero and the investment option treated as out-of-the-money. With the passage of time, the price trajectory can be expected to reach the cap P_H before reaching some level exceeding P_H, and since the value outcome $P_H Q/r$ is the same for both $P = P_H$ and $P > P_H$, there is no gain in waiting. The following analysis treats the threshold $\hat{P}_C$ as lying between the lower and upper limits.

When $P_L \le \hat{P}_C \le P_H$, the optimal solution is obtained from equating the investment option value with the active project net value at the threshold $P = \hat{P}_C$. The optimal solution is determined from both the value-matching relationship:

$$A_{C0}P^{\beta_1} = \frac{PQ}{\delta} + A_{C21}P^{\beta_1} + A_{C22}P^{\beta_2} - K \tag{16}$$

and its smooth-pasting condition expressed as:

$$\beta_1 A_{C0}P^{\beta_1} = \frac{PQ}{\delta} + \beta_1 A_{C21}P^{\beta_1} + \beta_2 A_{C22}P^{\beta_2} \tag{17}$$

when evaluated for $P = \hat{P}_C$. This reveals that:

$$\frac{\hat{P}_C Q}{\delta} = \frac{\beta_1}{\beta_1 - 1}K - \frac{\beta_1 - \beta_2}{\beta_1 - 1}A_{C22}\hat{P}_C^{\beta_2} \tag{18}$$

$$A_{C0} = \frac{K\hat{P}_C^{-\beta_1}}{\beta_1 - 1} - \left(\frac{1-\beta_2}{\beta_1 - 1}\right) A_{C22}\hat{P}_C^{\beta_2-\beta_1} + A_{C21}$$
$$= \frac{1}{\beta_1 - \beta_2}\left[\left(1-\beta_2\right)\frac{\hat{P}_C Q}{\delta} + \beta_2 K\right]\hat{P}_C^{-\beta_1} + A_{C21}. \tag{19}$$

The absence of a closed-form solution requires $\hat{P}_C$ to be solved numerically from (18), and A_{C0} from (19). Model 8, the investment option value $F_{C0}\left(P\right)$ for the project, is:

$$F_{C0}\left(P\right) = \begin{cases} A_{C0}P^{\beta_1} & \text{for } P < \hat{P}_C \\ \frac{PQ}{\delta} - K + A_{C21}P^{\beta_1} + A_{C22}P^{\beta_2} & \text{for } \hat{P}_C \le P < P_H, \end{cases} \tag{20}$$

where $P_L \le \hat{P}_C \le P_H$.

Figure 8 shows a spreadsheet for this investment Model 8, when Q=1.

Figure 8

	A	B	C	D
1			**INVESTMENT OPPORTUNITY FOR A PPP WITH A COLLAR OPTION**	
2	**INPUT**			EQ
3	P	6.00		
4	K	100.00		
5	σ	0.25		
6	r	0.04		
7	δ	0.04		
8	PL	**4**		
9	PH	**10**		
10	**OUTPUT**			
11	ROV CALL	**61.8978**	IF(B3<B13,((B4/(B14-1))*(B3/B13)^B14),B12)	7
12	P/δ-K	50.0000	MAX(B3/B7-B4,0)	5
13	P^	9.4279	(B14/(B14-1))*B4*B7	6
14	β_1	1.7369	0.5-(B6-B7)/(B5^2)+SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
15	A0	2.7547	(B4*(B13^-B14))/(B14-1)	8
16				
17	ROV COLLAR	**38.3688**	IF(B3<B20,B21*(B3^B14),B3/B7-B4+B23*(B3^B14)+B24*(B3^B18))	20
18	β2	-0.7369	0.5-(B6-B7)/(B5^2)-SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
19	FIND P^	0.0000	B20/B7-(B14/(B14-1))*B4+((B14-B18)/(B14-1))*B24*(B20^B18)	18
20	P^	4.0000	SOLVER Set B19=0, changing B20	
21	AC0	1.7862	(1/(B14-B18))*((1-B18)*(B20/B7)+B18*B4)*(B20^-B14)+B23	19
22				
23	AC21	-1.8520	($B$9/($B$9^B14))*(B26/B28)	15
24	AC22	112.2797	(-$B$8/($B$8^B18))*(B27/B28)	15
25				
26	[ ]	-0.0400	(B6*B18-B6-B7*B18)	15
27	( )	-0.0400	(B6*B14-B6-B7*B14)	15
28	{ }	0.0040	(B14-B18)*B6*B7	15
29				
30	AC21*P^β1	**-41.6129**	**B23*(B3^B14)**	PL<P<PH
31	AC22*P^β2	**29.9818**	**B24*(B3^B18)**	
32	ROV COLLAR	38.3688	B12+B30+B31	

From (18) the threshold $\hat{P}_C$ depends only on the floor P_L through A_{C22}, but not on the ceiling P_H. Adjusting the ceiling of the collar has no material impact on the threshold, so the timing decision is affected by the losses foregone by having a floor but not by the gains sacrificed by having a ceiling. Since A_{C22} is non-negative, the with-collar threshold $\hat{P}_C$ is always no greater than the without-collar threshold $\hat{P}_0$ (6), and an increase in the floor produces an earlier exercise due to the reduced threshold level.

Figure 8 shows that with a floor of 4 and ceiling of 10, and the other parameter values, the option coefficients A_{C21} and A_{C22} are -1.8520 and 112.2797 (15), so the ROV COLLAR is 38.4 (20) when $P_L<P<P_H$,

less than the ROV without collar 61.9 (7). Cell B32 shows that the ROV (COLLAR)=NPV(50)+ PUT(29.98)-CALL (41.61)=38.37. An investment opportunity with only a put is worth 50+29.98=79.98, and with only a written call 50-41.61=8.39. These values are also very sensitive to changes in the parameter values, as shown in the Appendix.

EXERCISE 14.2

Carlos Azevedo owns a solar plant, with a constant Q=1 KWh per year, the electricity price =€ 2, but the generous Portuguese government has guaranteed a revenue of € 4 per annum. If r=.04, electricity δ=.04, σ=20%, should Carlos try to sell this plant for €100, if A_{Cf11}=2.08, A_{Cf11}=133.33 ?

$$\beta_1, \beta_2 = \left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2} \right) \pm \sqrt{\left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}}$$

$$V_{Cf}(R) = \begin{cases} \dfrac{R_L}{r} + A_{Cf11} R^{\beta_1} & \text{for } R \le R_L \\ \dfrac{R}{\delta} + A_{Cf22} R^{\beta_2} & \text{for } R_L \le R, \end{cases}$$

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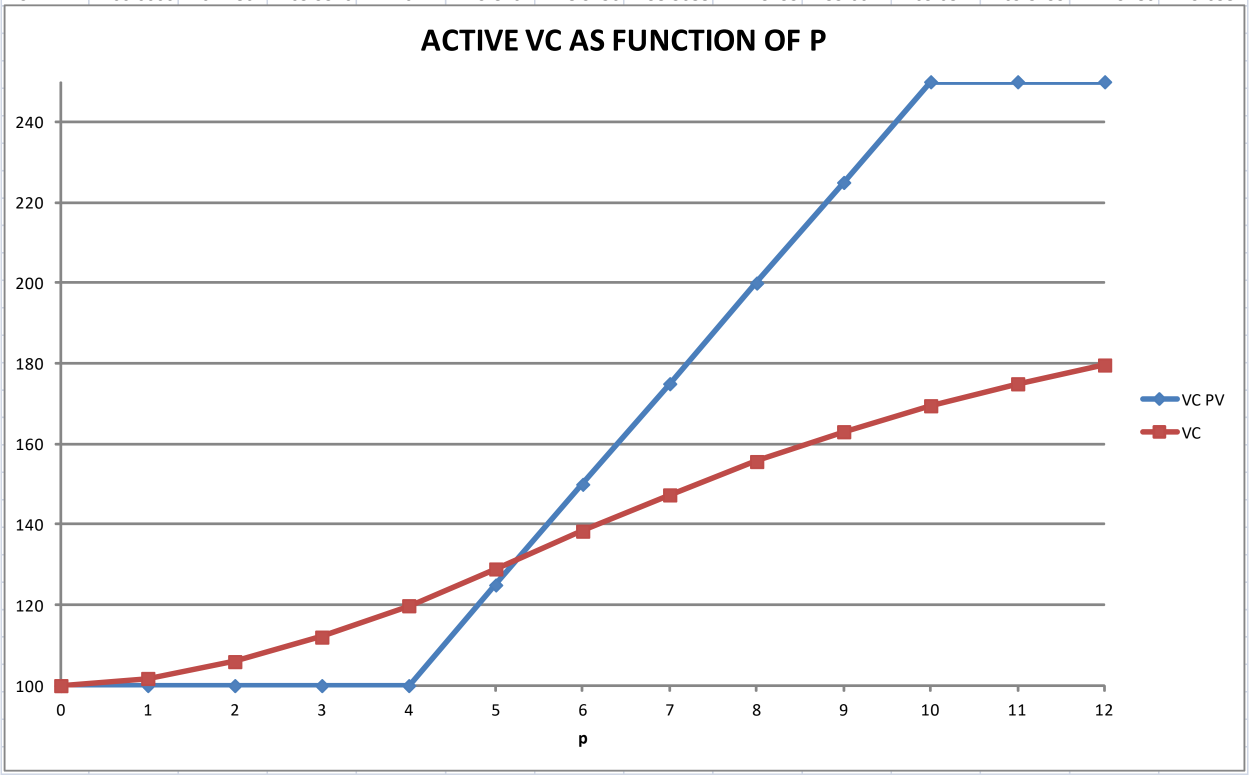
APPENDIX

Figure A2

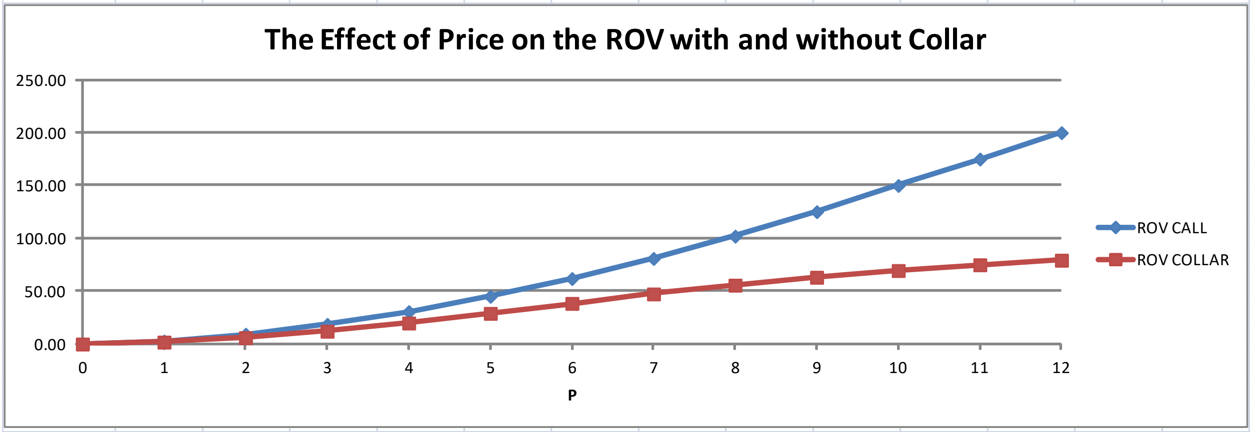
P	0	1	2	3	4	5	6	7	8	9	10	11	12
VC PV	100.0000	100.0000	100.0000	100.0000	100.0000	125.0000	150.0000	175.0000	200.0000	225.0000	250.0000	250.0000	250.0000
VC	100.0000	101.7862	105.9540	112.0411	119.8462	128.9756	138.3688	147.3733	155.6677	163.0814	169.5199	174.9786	179.6381



In Figure A2, past the floor price of P_L=4, the difference between the VC PV and the VC consists of a long position in a put option (should P go below 4) and a short position in a call option (should P rise above 10=P_H). If P=6, the net value of the put and call is negative, so the VC PV exceeds the VC. The (VC PV – VC) spread increases as P increases up to 10, the ceiling price.

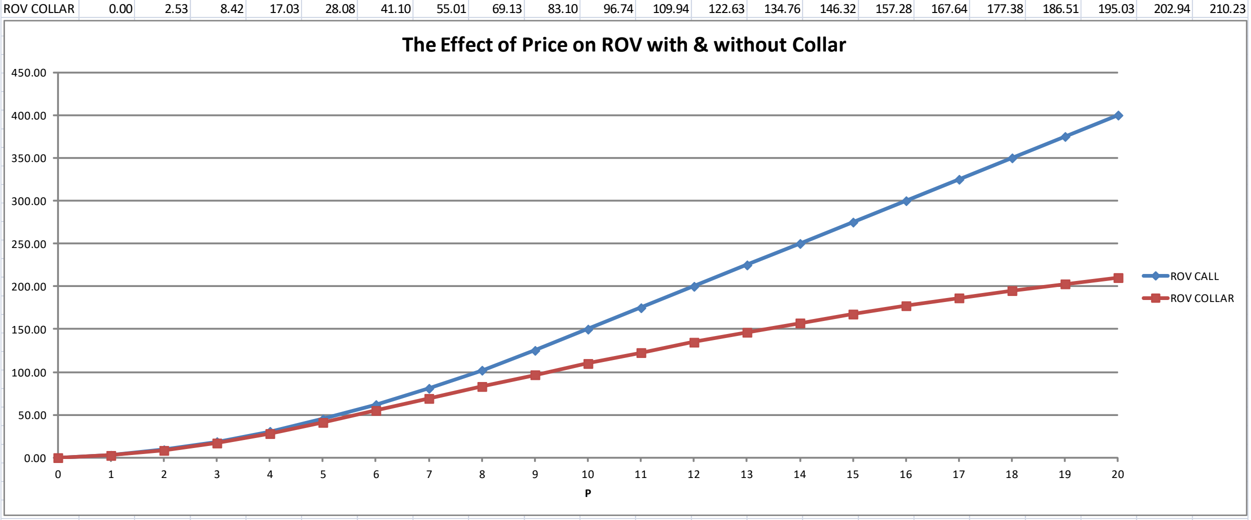
Figure A3

P	0	1	2	3	4	5	6	7	8	9	10	11	12
ROV CALL	0.00	2.75	9.18	18.57	30.61	45.10	61.90	80.90	102.02	125.18	150.00	175.00	200.00
ROV COLLAR	0.00	1.79	5.95	12.04	19.85	28.98	38.37	47.37	55.67	63.08	69.52	74.93	79.28



In Figure A3, the ROV Collar (P_L=4, P_H=10) always has a lower value than a standard ROV without a collar, since there is no upper limit to the investment profit, and the investment opportunity is an option, not yet a commitment. Figure A4

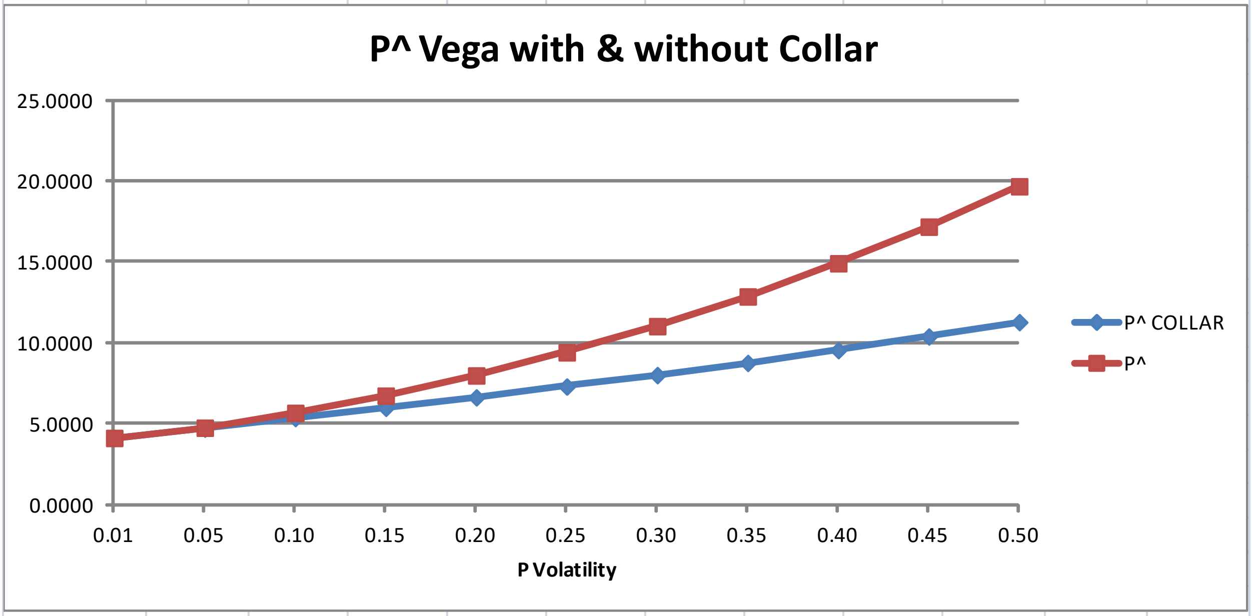
P	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ROV CALL	0.00	2.75	9.18	18.57	30.61	45.10	61.90	80.90	102.02	125.18	150.00	175.00	200.00	225.00	250.00	275.00	300.00	325.00	350.00	375.00	400.00
ROV COLLAR	0.00	2.53	8.42	17.03	28.08	41.10	55.01	69.13	83.10	96.74	109.94	122.63	134.76	146.32	157.28	167.64	177.38	186.51	195.03	202.94	210.23

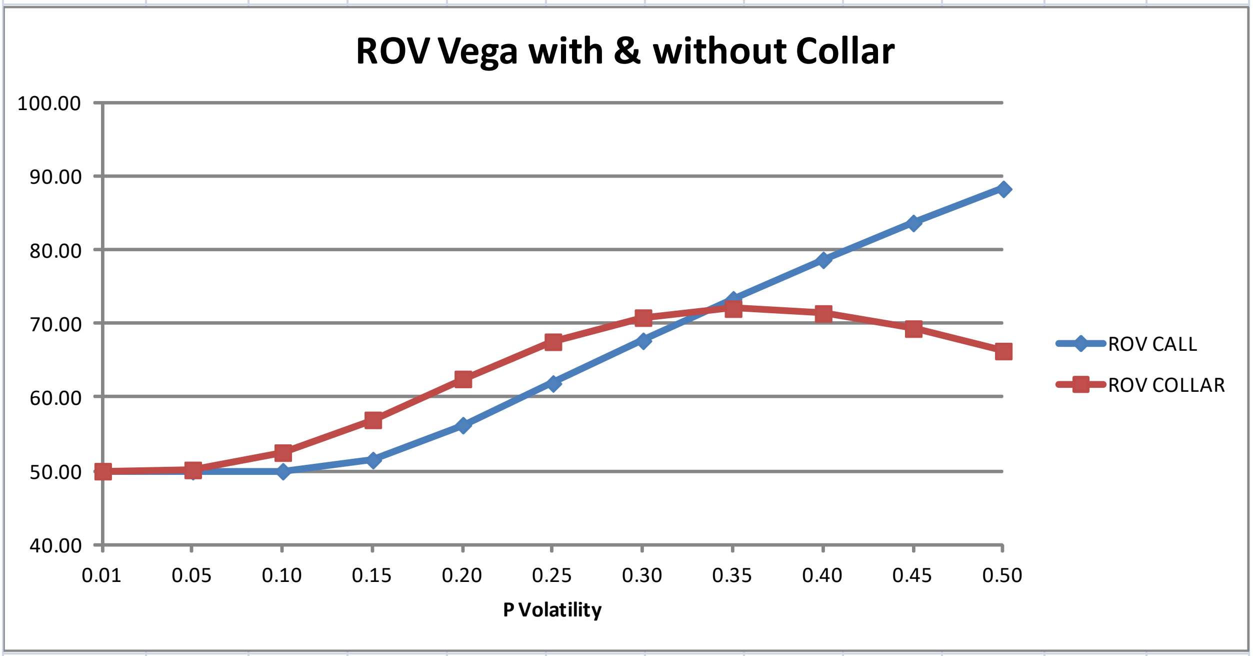


PL **4** PH **20**

In Figure A4, the ROV Collar with a higher price ceiling, in this case P_H=20, is more valuable than with the previous ceiling of P_H=10, and the spread between the ROV with and without collar increases as P approaches P_H. Figure A5

σ	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
ROV CALL	50.00	50.00	50.00	51.51	56.25	61.90	67.69	73.32	78.66	83.65	88.28
ROV COLLAR	50.00	50.18	52.52	56.97	62.51	67.55	70.82	72.03	71.40	69.34	66.30
P^ COLLAR	4.1439	4.7168	5.3681	6.0008	6.6458	7.3178	8.0254	8.7739	9.5670	10.4074	11.2970
P^	4.1439	4.7724	5.6861	6.7571	8.0000	9.4279	11.0523	12.8831	14.9282	17.1945	19.6873





P	2	PL	**3**						PH	**500**	

What is the affect of increasing volatility of the primary underlying factor on the threshold that justifies immediate investment, and also on the ROV (the so-called "vega"). Naturally the price threshold increases with the increased of expected price volatility shown in Figure A5 ($P=2$, $P_L=3$, $P_H=500$), so a government seeking early investment might consider imposing a collar in a volatile price environment. The ROV without a collar increases almost linearly with increases in the price volatility, but the ROV with a collar has a different pattern. From a low volatility environment, the ROV + Collar increases, but eventually at high expected volatilities the vega almost becomes negative, due to the increase in the value of the written call option.